Analogies in mathematical problem posing
Analogías en el planteamiento de problemas matemáticos

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Abstract

In this paper, we study two important aspects that characterize mathematical thinking: posing problems and the use of analogies. We start from a posing problem theoretical model in the school context, structured by six stages that are cognitively interconnected. In particular, through an experimental study the nature of the analogies in the posing problem process is analyzed. In order to achieve the former, we study the effect caused by the epistemic basis of instruction, the quality of the analogy and its localization in the cognitive framework. The empirical evidences suggest that in classroom, both the contextual as structural nature strongly respond to the encouragement of critical thinking.

Key words: *analogy, problem posing, problem solving, critical thinking.*

Resumen

Este estudio implica dos aspectos importantes que caracterizan el pensamiento matemático: el planteo de problemas y el uso de analogías. Se parte de un modelo teórico del proceso de planteo de problemas en el contexto escolar, estructurado por seis etapas que se interconectan en el plano cognitivo. Mediante un estudio experimental se analiza la naturaleza de las analogías en el proceso de planteo de problemas. Para ello se estudia el efecto que ocasiona la base epistémica de la instrucción, la calidad de la analogía y su localización en el esquema cognitivo. Las evidencias empíricas sugieren que, tanto la naturaleza contextual como la estructural, responden marcadamente al estímulo del pensamiento crítico en el salón de clases.

Palabras clave: *analogía, planteamiento de problemas, resolución de problemas, pensamiento crítico*

INTRODUCTION

Posing problems can be seen as a teaching activity that stands as a professional competence, associated with the elaboration of teaching tasks and to the graduation of its difficulty levels. Posing problems can also be seen as a learning activity where the student makes reasonable questions that express a higher understanding of mathematical contents (Leisen, 2006). This conception has been supported in many school curricula; therefore, posing problems is recognized as a necessary component of the teaching and learning of mathematics. As suggested by Kilpatrick, in math class, *"... problem formulating should be viewed not only as goal of instruction but also as a means of instruction. The experience of discovering and creating one's own mathematical problems ought to be part of every student's education"* (Kilpatrick, 1987, p. 123).

Researches carried out through the last years reveal several didactic, psychological and epistemological problems related to mathematics problem posing in the school. Usually, they explore the nature of posing problems, its inexhaustible potentialities inherent to thinking development in classroom, the structures of the psychological processes associated, the relations with problem solving and the connections with the creative thinking (English, Fox, & Watters, 2005; Leisen, 2006; Priest, 2009). When we are posing new problems as a task of the teacher is not clear what results are going to be obtained or even how it is going to take place or from what data the question will be formulated.

On the use of analogies in math, distinguished mathematicians have highlighted the importance of this logical form of thinking in connection with mathematical creativity. For instance, referring to the mathematical discovery, Poincaré points; *"Discovery is discernment, selection. [...] Mathematical facts worthy of being studied are those which, by their analogy with other facts, are capable of conducting us to the knowledge of a mathematical law, in the same way that experimental facts conduct us to the knowledge of a physical law" (Poincaré, 1914, p. 51). Polya, meanwhile, highlights the utility of using analogies for plausible reasoning as a kind of similarity that is singularized by the individual: "The essential difference between analogy and other kinds of similarity lies, it seems to me, in the intentions of the thinker" (Polya, 1954, p. 13).*

The study of posing problems, as a cognitive process, has been object of researches in math education during the last years (Christou, Mousoulides, Pittalis, Pitta-Pantazi, & Sriraman, 2005; English, Fox, & Walter, 2005; Priest, 2009; Kar, Özdemir, Sabri Ipek, & Albayrak, 2010; Yuan & Sriraman, 2011; Rosli, Capraro, & Capraro, 2014). Analogically, but in a relatively separated field, the study of analogical reasoning has also motivated the interest of several researches (Bernardo, 2001). As usual, the intersection of these two fields demarcates a less explored area from the empirical and theoretical points of view. The use of analogies during posing math problems defines a borderland, whose study can be beneficial to dig a little deeper into the complex cognitive processes associated with mathematical thinking. This research has the purpose of determining what place the analogies take during the process of posing mathematical problems and its possible connections with the epistemic bases that support the kind of instruction.

Theoretical and empirical background

Posing problems is a complex and multifactorial activity. From the didactic field, this concept refers both to the generation of new problems as to the reformulation of given problems. Posing problems is an important component of the professional abilities of a teacher. That is the more common perspective and reflects a predominantly teaching focus. However, if posing problems is carried out by students, then, from a less common perspective, this reflects a predominantly learning focus.

From the psychological field it is possible to assert that solving problems is the successive reformulation of an initial problem. It is natural that the process of formulating problems is associated with problem solving in a dimension that requires much creativity. Actually, it is said that the best results occurs in a problem solving environment (Kar, Özdemir, Sabri pek, & Albayrak, 2010). Posing problem requires productive thinking, that is why it has been very useful to measure fluency, flexibility and originality in the thinking of individuals (Sternberg & Sternberg, 2012). There is consensus that the generation of diverse and heterogeneous questions characterizes creative individuals (English, Fox, & Watters, 2005).

The mathematical expression of students through self-created problems reveals not only their level of comprehension and conceptual development, but also their perception about the nature of mathematics and their attitude toward that discipline (Priest, 2009; Lavy & Shriki, 2010). Therefore, a development in education that promotes critical thinking and a dynamic conception of mathematical knowledge is required. In general, the study of posing mathematical problems entails a context, a result and a process. The context is related with instruction, the result is the result is expressed in the new problem and the process subsumes the nature of the creative insight.

Posing problems acquires special features in the formation of a math teacher, that is why researches has been conducted both on in-service teachers (Peled, 2007; Chen, Van Dooren, Chen, & Verschaffel, 2011) as on prospective teachers (Crespo, 2003; Abu-Elwan, 2007; Crespo & Sinclair, 2008; Kar, Özdemir, Sabri pek, & Albayrak, 2010; Lavy & Shriki, 2010). In this context, special abilities for posing and solving problems are required. They even converge, at the same time, linking the training development for teaching and the need of an active and inquisitive learning of mathematics.

On the one hand, a math training that favors posing new problems is needed; on the other hand, an educational training to ensure the proposal of problems of different levels of difficulty for prospective students is required. The NCTM highlights the role of the teacher at the selection, modification and implementation of teaching tasks; *"By analyzing and adapting a problem, anticipating the mathematical ideas that can be brought out by working on the problem, and anticipating students' questions, teachers can decide if particular problems will help to further their mathematical goals for the class"* (NCTM, 2000, p. 53). Although preservice mathematics teachers normally have more interest in their science subject and students' cognitive development, this is not safe to take for granted that they are capable of posing appropriate problems in the school context.

It is not only important to formulate questions rigorously and provide problems with mathematical sense, but also to anticipate possible answers and be able to change the cognitive demands through reformulating the task. Recent researches indicate that most of the questions the teachers formulate are closed and factual and focused on memorizing and reproducing low cognitive demands procedures (Crespo & Sinclair, 2008). For prospective schoolteachers is complicated to pose problems that promote mathematical reasoning, taking into account that for many years of training they have been exposed to traditional questions posed by their teachers or from books (Crespo, 2003).

As a cognitive process, posing problems implies the execution of complex strategies, the deployment of abilities and math skills, the activation of metacognitive resources, the influence of beliefs and underlying conceptions (Chen, Van Dooren, Chen, & Verschaffel, 2011). The act of posing problems embodies specific techniques that have been identified by several authors: accepting/what-if-not? (Brown & Walter, 1990; Lavy & Bershadsky, 2010; Wang & Liu, 2008), what-if-more (Jim Kaput, 1984; cited by Kilpatrick, 1987), analogies and generalization/specialization (Polya, 1957), and many more. These techniques are part of more general strategies that have been described by the scientific literature.

Brown & Walter (1990) recognize the presence of four stages in the insight of posing problems: choosing a starting point, listing attributes, asking "What-if-not?" question asking, and analyzing the problem. Polya (1957) observed something similar in the process of solving math problems, four general stages (understanding the problem, devising a plan, carrying out the plan and looking back) describe the path that the solver follows. The identification of stages, both in posing as in solving problems, reflects the structural nature of the two cognitive processes and the close relation between them.

While the strategy "What-if-not?" becomes clear through numerous examples provided by Brown & Walter, these authors point out that the posing problem process is not linear and includes a sort of cycle (Brown & Walter, 1990, pp. 55-60). Based on this strategy, Wang & Liu (2008) have studied posing problems in analytic geometry and they have observed that students tend to transform the problem (the data or the questions) without keeping the consistency of its attributes, which leads to invalid problems. The empirical evidences do not show regressive actions facilitating metacognitive control during the process of posing, it means, students regularly do not implement an analogical process to "looking back" described by Polya (1957, pp. 14-16).

Christou, Mousoulides, Pittalis, Pitta-Pantazi, & Sriraman (2005) establish a taxonomy of the posing problems process, their model consists of four cognitive stages: comprehending, translating, editing and selecting. Because of their research on six grade students, they suggest that the process of posing problems start with comprehension, although there is a certain propensity for editing and selecting quantitative information, which is similar to the usual tendency to execution in the solving problems process.

In a research about posing problems by the in training math teacher, Cruz (2006) establishes a metacognitive strategy to generate problems from a mathematic object; the research takes into account the cyclic regressions and the stages suggested by Brown & Walter (1990). This strategy involves six stages: selection, classification, association, search, verbalization and transformation. The first five stages express a lineal path, but when transformation is included, the process becomes more complex. From now on, this strategy will be called SCASV+T. Structurally, this strategy (see Figure 1) begins with selecting of an given object o phenomenon, which corresponds with "*choosing a starting point*" described by Brown & Walter (1990) and expresses the intentionality of posing problem as a cognitive activity.

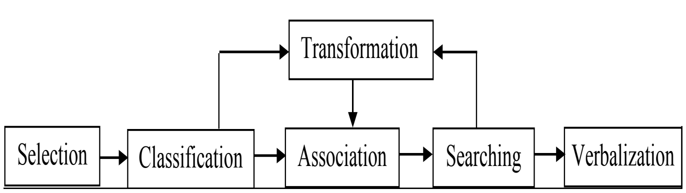

Figure1. A strategy for posing math problems.

When the subject disjoints the object or phenomenon through an analytic-synthetic process, which is similar to the heuristic strategy "decompose-recompose" described by Polya (1957) in the solving problem process. This second stage is called "classification", a mental operation that implies listing comparing and organizing attributes according to certain criteria (Inhelder & Piaget, 1969).

The next stage comprises the association of concepts with elements from the classification. Such elements can be linked with certain properties (area, perimeter, volume…). Through a process of decision making with the aim to speculate about possible relationships or dependences, the subject chooses a relatively small subset of such associated concepts. From this point of view, the stage of searching is analogous to the stage "devising a plan", pointed out by Polya (1957) in the process of solving problems, in both cases lies the enigma of insight. The last stage consists in the verbalization of the problem, which presupposes the organization and synthesis of the ideas. The problem or the conjecture allowing socializing of the problem is expressed through verbalization. This cognitive subprocess requires special communicative abilities, interconnected with the symbolic language of mathematics. No matter how automated it is, this stage involves difficulties with the rigor of the problem posed as well as with formulations that can be interpreted in different ways. It is important to differentiate the trivial making of a question from the conscious act of posing a problem. Verbalization is the materialization of posing and emerges as the result of a complex cognitive process, from an internal conflict related to an analyzed object or phenomenon.

If the path is not linear, it suggests that there are regressive subprocesses that return the information relatively differently with certain transformations. The stage of transformation acknowledges the freedom of the subject to make intentional changes during the strategy. Thus, it is reasonable to think that some components of the problem can be modified before, during and after the insight. From this point of view, the strategy "What-if-not?" may be located in the relations classification-transformation, or search-transformation. The trend to transformation can manifest in any stage of the cyclic structure, depending on the level of flexibility and criticism reached by the individual.

Some research has considered posing problems in an environment of criticism (Priest, 2009), while others have focused on posing problems as a skill or ability, (Abu-Elwan, 2007; Kar, Özdemir, Sabri □pek, & Albayrak, 2010; Chen, Van Dooren, Chen, & Verschaffel, 2011). Those are not divergent conceptions, but two views of the same construct from different angles. Dismissing notably traditionalist approaches (although not yet overcame) of math teaching, the following points of view are reflect of the conceptual complexity of posing problems in the school.

- Posing problems is a complex cognitive process, which presupposes a set of interconnected stages that can be automated. Therefore, teaching to pose problems requires the apprehension of those stages and the quality of the learning depends on the synergy of those interconnections
- Posing problems is the statement of a critical point of view of mathematics. The stages are not the content of the teaching, but the reflect of a critical and inquiring attitude of the mathematical knowledge

Both perspectives provide opportunities for the teacher to promote posing problems in the math class but, at the same time, generate some dangers. The first variant may result in the teaching of posing problem as some kind of algorithm, while the second warns about the inadequacy of the teaching method separated from the very conception of mathematical activity. If teaching emphasizes on the first approach then it maximizes the systemic character of mathematics; if it emphasizes on the second, maximizes the dialectic character.

The first approach allows that, the strategy SCASV+T may be seen as the structure of an ability that can be taught. This path has been proposed by Leontiev (1975) and leads to the searching for a guidance base for each action, to the determination of the most elemental operations of each stage, and to the automation of the systemic relationships that exist between them. Under this epistemology, learning can be seen as a training process by mental actions stages (Galperin, 1969), where the strategy requires of a series of internalization and automation moments until it becomes a habit.

The second approach is related to social constructivism, whose less controversial premise conceives the development of mathematics under the influence of language and social factors. However, the most controversial postulate conceives the mathematical knowledge under an analogic duality: on the one hand, necessary and autonomous; on the other (and at the same time), contingent, fallible and historically changing (Ernest, 1998). He has indicated, *"Mathematics consists primarily of human mathematical problem posing and solving, an activity which is accessible to all. Consequently, school mathematics for all should be centrally concerned with human mathematical problem posing and solving, and should reflect its fallibility"* (Ernest, 1998, p. 265). By taking from this epistemology the more stable path of critical thinking, the strategy SCASV+T would be an objective expression of a subjective will towards dialectic denial of the math content. Critical thinking requires motivation from the students' disposition (Barak, Ben-Chaim, & Zoller, 2007), and also includes a variety of abilities such as the identification of the information source, the analysis of its credibility, the reflection on the information consistency regarding previous knowledge, and the statement of conclusions.

The use of analogies in posing math problems

Before analyzing the place that analogies take in the process of posing problems, it is necessary to identify some methodological aspects adopted in the following study. An analogy can be described as a thinking mechanism, as a way of thinking and even as some kind of similarity. According to Vosniadou (1995), the essence of analogical reasoning lies in the identification and transfer of structures and relationships from a well- known system (from the source) to a less well- known system (to the target). The analogy requires maintenance, handling, activation and selective inhibition of mental representations, aimed to establish correspondences and inferences about similarity relations of higher order. Besides, analogic reasoning is configured thanks to various mental operations that are especially important in a wide sense of human cognition, such as comparison, analysis, synthesis, generalization, classification, and identification of cause-effect relationships.

Taking into account the volume of information transferred from the source to the target, analogies can be classified in two types: analogies between properties and analogies between relationships (Guetmanova, 1989). From this classification, it is possible to consider a simplified classification of math problems analogous at the school context. In effect, the posed problems can be structurally analogous because they contain data or similar questions (external analogy) or they can be analogous because of the variants solutions (internal analogies). As with other, the analogical reasoning changes by its meaning, function, range and nature. The learning through analogies lies in the visual perception, it means, the heuristic use of analogies is more supported on mental images than on logic prepositions of the reasoning. Reasoning using analogies, recognizing and transferring numerous structures between kinds of problems, promotes student performance at problem solving.

In the field of posing math problems, the researches about to the use of analogies are rare. For instance, in a study on the posing of word-problems, Bernardo (2001) establishes a strategy that promotes the analog transfer and facilitates the retrieval of relevant analogous problems. Meanwhile, Peled (2007) investigates the role of analogical reasoning on the design of tasks for math teachers' training. Taking into account the strategy SCASV+T, a step forward in that direction leads to the analysis of the place that analogies take during the implementation of the strategy. Therein lies the general motivation of this study, with the peculiarity that it takes into account the underlying epistemological base in the teaching method.

QUESTIONS, VARIABLES AND HYPOTHESIS OF RESEARCH

The main question of this study is. What is the nature of the use of analogies during the posing of math problems? This purpose implies at least three aspects to consider. In first place, the quality of analogical reasoning, this requires from certain criteria to facilitate its evaluation. Second, the positioning of those analogies in the process of posing, so also it requires a kind of a structural theoretical framework that facilitates the location in the posing process. Third, it is important to consider the environment where it takes place, which initially can be referred to the epistemic base underlying in the teaching method. Therefore, the overall question has been divided in three more concrete subquestions.

SQ1: What is the effect of the epistemic basis for instruction in the use of analogies during posing problems?

SQ2: Where are the analogies located in the framework of posing problems?

SQ3: Is there a connection between the epistemic basis for instruction and the location of the analogies during posing problems?

Three variables to concrete the study are established: Epistemic Base for Instruction (EB), Location of the Analogies (LA) and Quality of Analogies (QA). The variable EB entails three variants: one based on an environment of criticism where posing problems is a natural aspect, other based on the conception of posing problem as an ability, and a third that entails the traditionalist perspective in which posing problems is subjected to eventuality. The variable LA is focused on the strategy SCASV+T as a structural framework, so that the location of the analogies may be conceived in a more simplified way. In this case, four possible variants of localization are identified: inside the cycle association-search-transformation, in and out of this cycle, out of the cycle, and the 'no observation' of analogies. Finally, to specify the values of the variable QA, the following indicators are established:

- Skills to transfer related structures from the source to the target (in the sense Vosniadou 1995)
- Abilities to verify true analogies and to discard false analogies (taking into account the connections with problem solving English, Fox, & Watters, 2005; Leisen, 2006; Priest, 2009).
- Variety of analogies (analogies regarding the structure of the problem, analogies regarding solutions, and both; similar to the classification of Guetmanova 1989).
- Mathematical complexity of the analogies (relating to the identification and transfer of numerous structures between mathematical objects Wilbers & Duit, 2006).

The ordinal value of each indicator is proposed by the criteria of the evaluator, following the scale 0, 1, 2. Particularly, in the case of the third factor (variety of analogies) the values are assumed as follows: 0 = the use of analogies is not observed, 1 = one kind of analogy is exclusively used, 2 = both types of analogies are used together. The sum S of all the indicators range in the next range $0 \leq S \leq 8$, however the evaluator can add or discount a point depending on the originality of the problem or of the mistakes committed respectively.

From the sum S, the final assessment of the variable QA has five possible values, as detailed in Table 1. For instance, the quantitative value QA=3 synthesizes a set of qualitative variants positioned on the middle level, as a high mathematical complexity but without progress, a rigorous and analogical thinking but with low math complexity, between other variants.

Table 1. Variables of research

Variable	Values	Description
EB: Epistemic Base for Instruction	PVI	The teaching method is based on an inquisitive point of view of math problems (critic retrospective questions are made after solving each problem and the possibility of posing new problem or to reformulate the problem is analyzed).
	PPH	The teaching method conceives posing problems as an ability that can be formed by relatively stable stages (the mental actions that structure this ability are object of teaching).
	MET	The teaching method is traditional, in the sense that emphasizes on posing problems and is subjected to eventuality.
LA: Location of the Analogies	DeC	Inside the cycle association-search-transformation
	TaC	In and out of the cycle.
	FuC	Out of the cycle.
	NoA	No observation of analogies.
QA: Quality of Analogies	1	Very low ($S \leq 0$)
	2	Low ($1 \leq S \leq 2$)
	3	Average ($3 \leq S \leq 5$)
	4	High ($6 \leq S \leq 7$)
	5	Very high ($S \geq 8$)

To study each subquestion the following null hypothesis are stated:

H1: There is no meaningful association between the Epistemic Base for Instruction (EB) and the Quality of Analogies (QA) at posing math problems.

H2: There are not meaningful differences between Location of the Analogies (LA) and In and out of the cycle association-search-transformation.

H3: There is no meaningful association between the Epistemic Base for Instruction (EB) and the Location of the Analogies (LA).

METHODOLOGY

Subjects and context

The teaching of the strategy SCASV+T is not viable in the school context since posing problems is not part of the curriculum and is barely visible in the formulation of the objectives. However, the curricula of the math teacher in training are more flexible and enable the incorporation of this aspect as an object of learning. Even more if we take into account the need to build professional skills, to design exams, to exemplify and to motivate interest for new contents. For that reason, the study is carried out in the Bachelor of Education, specializing in Sciences. The sample consists of 64 preservice teachers enrolled in the second year of this bachelor at three universities of pedagogical sciences in the western region of Cuba. At the moment of the study, all the students had recently returned to teaching at their respective universities, after their pre-professional training in high schools for over a month. The three professors involved have more than ten years of experience at teaching the discipline Methodology of Teaching Mathematics, especially in solving problems at school context. Each teacher has an assistant who collects the data and processes the information.

Treatment

Since there is no random selection of the members, the three groups this research uses are complete units. Each group is in a different university and they are used as are organized. A quasi-experimental design as shown in Table 2 is applied, which is an "extended nonrandomized pretest-posttest control group design" utilized by some authors. The groups are randomly assigned to a treatment condition, according to the variable EB: two experimental groups (EXP_1, N = 21, BE = PVI; EXP_2, N = 20, EB = PPH) and a control group (CONTROL, N = 23, EB = MET). The observation of the dependent variables is made before and after the treatment administration. The matching-only pretest-posttest is outlined in Table 2.

Table 2. Design of the study

Group	N	Pretest	Treatment	Posttest
EXP_1	21	LA, CA	PVI	LA, CA
EXP_2	20	LA, CA	PPH	LA, CA
CONTROL	23	LA, CA	MET	LA, CA

The treatment had simultaneous actions in all three groups and specific actions in the experimental groups as described as follows:

Simultaneous interventions on the three groups:

1. The research takes three months (the time that the topic Teaching of Geometry consumes, whose development is planned simultaneously in the three pedagogical universities).
2. The authorization of the competent authorities was asked to modify the syllabus of the subject Methodology of Teaching Mathematics considering the absence of objectives related with posing problems, This is possible thanks to the flexibility of the syllabus D.
3. A first meeting with the teachers of the three groups where the purposes of the research are explained is conducted. All teachers consider this study important and agree to collaborate.
4. The software Cabri Géomètre is used, which, after completion of construction, allows the user to freely move items by dragging them, so it is possible to observe how other elements dynamically respond when the initial conditions are changed. This favors a dynamic environment, where the search of relations and dependencies leads to the formulation of conjectures. For instance, when certain elements are moved, others seem to remain fixed, or simulate a circumference and provide countless possibilities to imagine hypotheses that later will have to be verified or discarded.
5. Two special practical classes are held (one at the beginning and one at the end of Teaching of Geometry), which constitute an adequate environment for posing geometric problems as an activity of the professional training. These spaces are used to assess the dependent variables in this study.

Simultaneous interventions on both experimental groups:

- A special conference entitled "*Posing Geometrical problems at High School*" is developed, where the didactic value of posing new problems is explained. The strategy SCASV+T is presented too as a viable path for posing problems, which can foster a creative behavior.
- In a practical class the use of analogies, the technique generalization/specialization and the strategy "What-if-not?" to pose geometric problems are exemplified. The practice is developed through the strategy SCASV+T and with the assistance of the software Cabri Géomètre.
- The students are invited to present their own thinking strategies in front of their classmates, so they can share the thinking path that leads them to create new problems. This process is difficult given the natural fear to open the mind in front of an audience. At the beginning, the teachers illustrate how they imagine new problems.

Specific interventions in the experimental group EXP_1:

1. A critical environment on the creation of math problems is established. The tasks of solving problems are combined with tasks oriented to encourage the posing of new problems.
2. The future teachers design new problems, aimed to apply them in school. In small groups the effectiveness of the new problems to assess learning is discussed, to motivate new contents, to address the development of heterogeneous cognitive levels, among other didactic actions.

Data

In general, scientific literature has few empiric instruments to investigate the path of reasoning during the posing of math problems. This study implements two instruments jointly before and after treatment. The assessment of the dependent variables is done qualitatively integrating the results of both instruments, according the agreed criteria of the professor and his/her assistant.

Instrument No. 1. Special tasks to assess posing problems are designed (Silver & Cai, 2005), which involves four stages. At the first stage, the subjects have to pose problems related to a specific given geometric object. At the second stage, they have to try to solve their own problem. At stage three, they are allowed to make modifications to the problem or to the geometric object, not only expecting to create a more challenging idea than the original but also with the purpose of rectifying mistakes at posing the problem, that were discovered while trying to solve it (stage four).

Instrument No. 2. For a deeper study of the cognitive activity, it use an instrument described by Cruz (2006) on the field of problem solving. At the vertical axis is placed a nominal scale that contains the stages of the strategy SCASV+T; while on the horizontal axis is placed the elapsed time. The Cartesian product gives a graphic episode that illustrates the behavior of the activity of the subject during the process of posing problems. For more objectivity, the oral act of creation of a new problem in front of the blackboard is recorded and then analyzed in other work session.

RESULTS AND DISCUSSION

Although to some extent both instruments base their findings on introspection, they provide of objective information about the process of posing problems. The first instrument was easy to apply, because often the assessment of the learned is through written work. The most common was that the problems did not have full connection between what is given and what is searched, essentially, due lack of data and contradictions between them. When passing to the second stage, where the student is asked to solve the posed problem, the subjects often realized of the mistakes made at posing. For that reason, the making of transformation allowed not only to correct the problem but also to develop professional abilities. At the last stage, the students tended to forget how they have imagined the problem. Therefore, they were allowed to review the worksheets of the first three phases.

The second instrument was designed to record the information and for its implementation another work session was carried out and it was private. In this case, each student was requested to reproduce on the board what had happened during the implementation of the first instrument. The Figure 2 shows a graphic episode of the process of posing a geometric problem. The first three minutes offer an idea of what happened at the first stage of work, where there was a tendency to pose in a linear path (without transformations) that led to a badly formulated problem. When dialoguing, the student quickly explains where the error is, due that he has already detected it while trying to solve the problem for the first time. He immediately explains the transformations that had to do and recalls a similar situation he knew of another previously solved problem.

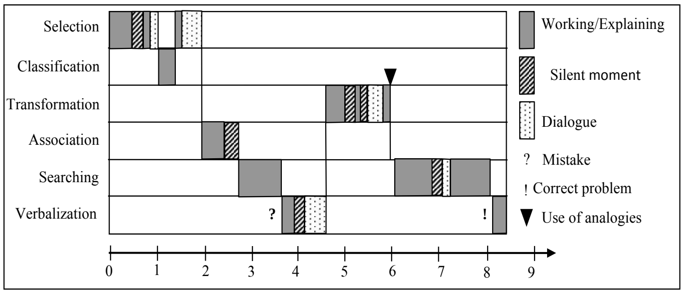

Figure 2. Graphic episode of the posing of geometric problem

Although the analogy is established after the attempt of solution, it does not refer to the solution but to the problem structure. As a result the subject makes some changes to the figure of the original problem. Therefore, from the sixth minute, the explanation the student gives about the searching of relations and dependencies can be perceived. In these cases, he evades possible complications and ends making a question relatively simple, similar to the problem he remembered. While the ultimate problem is quite trivial, it expresses a tendency to rigor because the subject tries to be more cautious after committing an error. The former can be motivated by the fear of failure, but it leaves a favorable impression on the sense of criticism accompanied by analytical reasoning

To assess the student's performance with variable QA, there was agreement on giving the following grades by indicator: skill = 1, ability = 2, variety = 1 (one kind of analogies), complexity = 0. All totalizes $S = 4$, without deciding adding or discounting an additional point. According the Table 1, since $3 \leq S \leq 5$, the quality of the analogy is assess as 3 (average).

From the obtained data, the result of the analysis of possible associations between the Epistemic Basis for Instruction (EB) and Quality of the Analogies (QA) when posing mathematical problems are presented. Table 3 resumes the results of the variable QA ($1 \leq QA \leq 5$; where theoretically QA is a function that depends of the EB treatment).

Table 3
Mean values (standard deviations) of the measurement of CA variable

Group	Treatment (EB)	Pretest	Posttest	DQA
EXP_1	PVI	1.86 (1.01)	2.81 (1.12)	.95
EXP_2	PPH	2.00 (1.17)	2.55 (1.15)	.55
CONTROL	MET	1.87 (.97)	1.91 (1.08)	.04

We test for differences among the tree treatment-groups (EXP_1, EXP_2 and CONTROL) on the posttest using an ANCOVA to control for group differences on the pretest. The choice of the pretest as a covariate is appropriate given the substantive relationship between the pretest and posttest scores and the strong Pearson correlation between them, $r(64) = .82$, $p < .01$. A necessary first step when using ANCOVA is to test the homogeneity of regression slopes assumption. In SPSS output, the source Treatment*Pretest is not statistically significant, $F(2, 58) = .49$, $p > .05$. This indicates that the factor (Treatment) and the covariate (Pretest) do not interact and, thus, the assumption of homogeneity of regression slopes is met.

After testing the homogeneity of regression slopes assumption, we proceed with ANCOVA to test for differences between the treatment groups in the posttest controlling for their pretest differences. The results from the Levene's test show that the assumption of homogeneity of variance is also met, $F(2, 61) = 2.40$, $p > .05$. The F-test for effect of Treatment shows that there are statistically significant differences between the treatment groups on the posttest scores when controlling for pretest groups differences, $F(2,60) = 14.84$, $p < .01$, $\eta^2 = .33$. As the omnibus ANCOVA test indicates statistically significant differences between the adjusted means of the three treatment groups, a post-hoc multiple comparisons test is needed. The results in Table 4 show that all differences between the adjusted posttest means of each experimental and control groups are statistically significant. The F-test allows us to conclude that $QA_{(MET)} < QA_{(PPH)} < QA_{(PVI)}$, which allow to reject the null hypothesis H1. Hence, there is a significant association between the Epistemic Base for Instruction (EB) and the Quality of the Analogies (QA) in mathematical problem posing.

Table 4. Multiple comparisons for adjusted posttest group means using pretest as covariate

Contrast comparison	ΔM	SEΔM	$df_{contrast}$	df_{error}	F	95% CI for ΔM	
EXP_1 vs. EXP_2	.39	.17	1	60	5.09^{*}	.04	.74
EXP_1 vs. CONTROL	.91	.17	1	60	29.37^{**}	.57	1.24
EXP_2 vs. CONTROL	.52	.17	1	60	9.23^{**}	.17	.86

Note. ΔM = Difference of adjusted means. SEΔM = Standard error of ΔM.* p < .05.**p < .01.

Although the changes seem small, the increases in the variable QA in both groups are substantial compared with the control group. The increase in EXP_1 is approximately $\Delta QA = 2.81 - 1.86 = 0.95 \approx 1$, which almost is a change in quality from Low to Average. The four indicators of quality did not progress evenly for the most students. Particularly, the last two indicators (variety and complexity) did not change significantly in this study. The most frequent analogies focused on the solutions of the problems and not on towards the structure of the problems. In addition, reducing the degree of difficulty was an elusive way to accomplish the task of posing new problems, mainly due the increase of initial data.

For the study of localization of the analogies (variable LA) the three groups were examined as a unique sample (N = 64) at the beginning of the experiment. Taking into account that the hypothesis H2 is outside the experimental treatment and considers the localization of the analogies in a natural environment, the pre-test was considered exclusively. Using a χ^2 goodness-of-fit test, the LA values differ significantly across the uniform distribution ($\chi^2(3) = 20.13$, $p < .01$). Categories DeC and FuC were respectively the more and the less frequent. The correspondent standardized residuals (SR = 3.5 and -2.5) exceed 2.0 in absolute value. This indicates that the differences between the observed and expected frequencies in both categories are a major contribution to the statistical significance of χ^2 value. This analysis leads us to reject the null hypothesis H2. Hence, there is a significant difference between the Localization of the Analogies, into and out to the cycle association searching transformation. The results are summarized in the following Table 5.

Table 5. Frequencies of LA variable considering the three groups as an only sample

Categories	Observed frequency	Expected frequency	Standardized residuals (SR)
DeC	30	16.0	3.5*
TaC	17	16.0	.3
FuC	6	16.0	-2.5*
NoA	11	16.0	1.3

Note. The *SR* values with asterisk (*) are statistically significant as they exceed 2.0 in absolute value.

Finally, the analysis of the results ends with the identification of relations between the values of LA before and after the treatment, considering EB as a stratification factor. The Table 6 shows the tabulated values and corresponding standardized residuals *SR*. There can be seen some values of *SR* whose absolute values exceed 2.0, suggesting the existence of certain apparent patterns. For instance, considering the values marked with an asterisk, the first stratum EXP_1 suggests an interesting kind of symmetry (TaC ⇆ FuC), while the absence of analogies seems to be stable (NoA ⇆ NoA).

Table 6
Observed values (standardized residuals) in pretest-posttest cross tabulation of LA variable

Treatment DeC TaC			Posttest				Total
			FuC	NoA	InC		
EXP_1	Pretest	DeC	8 (1.0)	2 (-.2)	0 (-1.0)	0 (-1.0)	10 (2.1*)
		TaC	2 (-.5)	1 (-.2)	2 (2.2*)	0 (-.7)	5 (-.1)
		FuC	0 (-1.1)	2 (2.2*)	0 (-.4)	0 (-.4)	2 (-1.4)
		NoA	2 (-.2)	0 (-1.0)	0 (-.6)	2 (2.6*)	4 (-.5)
	Total		12 (2.9*)	5 (-.1)	2 (-1.4)	2 (-1.4)	N = 21
EXP_2	Pretest	DeC	5 (.1)	2 (.3)	0 (-.9)	1 (.2)	8 (1.3)
		TaC	4 (-.1)	1 (-.3)	2 (1.6)	0 (-.8)	7 (.9)
		FuC	1 (.5)	0 (-.4)	0 (-.3)	0 (-.3)	1 (-1.8)
		NoA	2 (-.3)	1 (.2)	0 (-.6)	1 (.9)	4 (-.4)
	Total		12 (3.1*)	4 (-.4)	2 (-1.3)	2 (-1.3)	N = 20
CONTROL	Pretest	DeC	5 (-.1)	4 (.9)	1 (-.5)	2 (-.4)	12 (2.6*)
		TaC	3 (.6)	1 (-.1)	1 (.4)	0 (-1.0)	5 (-.3)
		FuC	2 (.6)	0 (-.8)	0 (-.6)	1 (.4)	3 (-1.1)
		NoA	0 (-1.1)	0 (-.8)	1 (1.0)	2 (1.7)	3 (-1.1)
	Total		10 (1.8)	5 (-.3)	3 (-1.1)	5 (-.3)	N = 23

Note. The *SR* values with asterisk (*) are those that they exceed 2.0 in absolute value.

A chi-square test of homogeneity was performed to determine the truthful associations. Only the first layer (EXP_1) produces significant values, contrasting LA variable in pretest against posttest. The results were complemented using symmetric and directional measures. The contingency coefficient (*C*) is a symmetric measure that attempt to quantify the strength of the relationship. In all cases $C \geq .3$ which is an evidence of a strong connection, but only the first is statistically significant ($\chi^2(9) = 23.17$, $p < .01$). Using the Goodman and Kruskal's tau as a directional measure, we found two issues. Firstly, in the group EXP_1 there is a bidirectional and significant dependence between the values of LA variable, before and after the treatment (accurately a 34.7% and 33.2% reduction in misclassification). Secondly, in both remaining groups all measures report small and non-significant values, indicating that the association is almost isolated. This information is summarized in Table 7.

Table 7. Chi-square test of homogeneity complemented by symmetric and directional measures

Treatment	χ^2 (df = 9)	Contingency coefficient (C)	Goodman and Kruskal's tau	
			LA dependent in pretest	LA dependent in posttest
EXP_1	23.17**	.72**	.35*	.33*
EXP_2	6.28	.49	.14	.07
CONTROL	9.98	.55	.13	.15

Note. $^{*}p < .05$. $^{**}p < .01$.

When these results are taken into consideration, we would be able to reject the null hypothesis H3. However, recurring to the first layer in Table 6, we can see that 15 cells (93.8%) have expected count less than 5. It is well-known that an approximation to chi-square distribution is usually satisfactory provided that expected frequencies in all the cells are at least as large as 5. Consequently, evidences about an interconnection between variables EB and LA are not conclusive in this study:

1. the studied sample consists of prospective teachers, who are subjects of advanced cognitive development and with some motivation for mathematics which reduces the level of generality.
2. the study used the actions of the strategy SCASV+T as a theoretical model of the process of creating new problems, which still requires more precision from the empirical point of view.
3. the instruments used and the measurement scales have not been adequately standardized in terms of reliability and validity, although their use has been positive to obtain information about the structure of the posing process and the localization of analogies.

In both experimental groups, the strategy SCASV+T was incorporated as learning objective; besides, the approach followed was predominantly didactic and oriented towards the development of professional competencies. The strategy stages and the fabric of its relations constitute the underlying pattern to explain the process of posing new problems. A different path may consist in modeling the process of posing from the psychological point of view, but there is the risk of leaving the process of creating new problems to spontaneity. To go deeper in the knowledge of the structures and functions of this cognitive activity a dialectic unit between the categories formation and development is needed.

The process of problem posing can be taught through strategies, to which particularities individuality of the subject. It means the guidance base on six actions whose interconnections were previously exemplified, but in the internalizing process each individual configures their own strategies. The strategy SCASV+T was merely a plausible path to find new problems; therefore, the analysis focused on the way that each individual carried out the internalization, depending on the degree of criticism in the classroom.

On the other hand, the dependent variables studied do not cover completely the complexity inherent of the nature of posing mathematical problems. There are other important aspects that have not been included in this study, as the presumed relation between posing and solving problem. Related to the former, a post-hoc analysis of experimental results revealed that students with more aptitude towards solving problems tended to pose more complex problems, to formulate questions consistent with the data and to spend less time at the stage of searching relations and dependencies. The students, with lower academic achievement, behaved in a way relatively different; so that it may be concluded that it is a manifestation of the close relationship between posing and solving problems. However, such conclusion is merely apparent.

In the process of posing problems, the evidences suggest that the quality of the analogical processes and the dynamic of their location are relatively dependent on the level of criticism of the subject. This criticism is interconnected with conscious retrospective analyses, and softens the tendency, already pointed out by Wang & Liu (2008), to transform the problem without keeping the consistency of the attributes, which was rarer in the experimental group EXP_1. Taking in consideration the observations from Wilbers & Duit (2006), the tendency to make analogies in the cycle association searching transformation suggests that the connections with mental images are activated during the most dynamic part of the process: the cycle association searching transformation. In experimentation, it was interesting the attention of some subjects showed regressive conducts from the searching stage towards the association stage. That can be explained in the sense that they went through the stage transformation automatically. However, regressive conducts toward the stage classification were sporadically observed which highlights the need to continue enriching the SCASV+T strategy from the structural and functional points of view.

CONCLUSIONS

As a first approach, the nature of the analogies has been analyzed from three perspectives: the first concerns the learning environment that encourages it implementation during the posing of problems, the second is related with its location in the structural framework of the cognitive process, and the third referred to the inherent qualities of the analogies. Since the dependent variables point to the quality of analogies and its cognitive localization, two main conclusions of this research can be extracted.

An enabling environment that promotes the use of analogies is learning where posing problems manifests itself consciously and is expressed intentionally. It is not enough incorporating posing problems to the curricular goals and even to determining or to ensure the framing basis of the mental actions that take place. It is needed incorporating a curious conception of the mathematical knowledge to the class, which, besides being a challenge for a teaching and learning of a developer way of the math.

As a psychological process, posing problems entails a set of interconnected stages, where a higher degree of complexity of the interconnections reflects a higher level of development of the ability to pose problems. The identification of these stages is useful to localize the position of the analogies, which tends to happen at the cycle association searching transformation, with a tendency to accentuate in a context that encourages critical thinking.

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The trailer of science in informal education in Jalisco, México

El tráiler de la ciencia en la educación no formal en Jalisco, México

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Abstract

As a strategy to strengthen the education in Jalisco, mainly with elementary school students and teachers, and social adaptation of science, technology and innovation, through itinerant trailers workshops and exhibitions of scientific experiments are developed in a fun way. This strategy of social adaptation of science has had a significant social impact, facilitating the approach of science, not only for children who are the target population, but for various social groups (like housewives and workers), several of whom are not attending or participating in typical activities of public presentations of science. Furthermore, trailers of science have served as a bridge of communication in regions that do not have options that are specific to large urban areas such as interactive science museums and the options offered by universities and research centers within their academic facilities.

Key words: *trailer of science, non-formal education, science communication, social appropriation of science, science for kids.*

Resumen

Como una estrategia para fortalecer la educación de Jalisco, principalmente con alumnos y maestros de educación básica y de apropiación social de la ciencia, tecnología e innovación, mediante los tráileres itinerantes de ciencia se desarrollan talleres y exposiciones de experimentos científicos de forma lúdica. Esta estrategia de apropiación social de la ciencia ha tenido un notable impacto social, al facilitar el acercamiento de la ciencia, no sólo a los niños –población meta, pero no exclusiva-, sino a grupos sociales diversos (como amas de casa y trabajadores), varios de los cuales no son asistentes ni participantes comunes en actividades típicas de divulgación de la ciencia. Asimismo, los tráileres de la ciencia han servido como un puente de comunicación en aquellas regiones que no cuentan con opciones que son propias de las grandes áreas urbanas, como los museos interactivos de ciencia y las opciones ofrecidas por universidades y centros de investigación dentro de sus instalaciones académicas.

Palabras clave: *tráiler de la ciencia, educación no formal, comunicación de la ciencia, apropiación social de la ciencia, ciencia para niños.*